

$\mathcal{N} = 2$ SCFT and M Theory on $AdS_4 \times Q^{1,1,1}$

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ABSTRACT

Coincident M2 branes at a conical singularity are related to M theory on $AdS_4 \times X_7$ for an appropriate 7 dimensional Sasaki-Einstein manifold X_7 . For $X_7 = Q^{1,1,1} = (SU(2) \times SU(2) \times SU(2))/(U(1) \times U(1))$ which was found sometime ago, the infrared limit of the theory on N M2 branes was constructed recently. It is the $SU(N) \times SU(N) \times SU(N)$ gauge theories with three series of chiral fields $A_i, i = 1, 2$ transforming in the $(\mathbf{N}, \overline{\mathbf{N}}, \mathbf{1})$ representation, $B_j, j = 1, 2$ transforming in the $(\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}})$ representation and $C_k, k = 1, 2$ transforming in the $(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N})$ representation. From the scalar Laplacian of X_7 on the supergravity side, we discuss the spectrum of chiral primary operators of dual $\mathcal{N} = 2$ superconformal field theory in 3 dimensions. We study M5 branes wrapped over 5-cycle of X_7 which were identified as (three types of) baryon like operators made out of N chiral fields recently. We consider M5 brane wrapped over 3-cycle of X_7 which plays the role of domain wall in AdS_4 . The new aspect arises when baryon like operators(M5 branes wrapped over 5-cycle) cross a domain wall(M5 brane wrapped over 3-cycle), M2 brane between them must be created.

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1 Introduction

In [1] the large N limit of superconformal field theories (SCFT) was described by taking the supergravity limit on anti-de Sitter (AdS) space. The scaling dimensions of operators of SCFT can be obtained from the masses of particles in string/M theory [2, 3]. In particular, $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory in 4 dimensions is described by Type IIB string theory on $AdS_5 \times \mathbf{S}^5$. This AdS/CFT correspondence was tested by studying the Kaluza-Klein (KK) states of supergravity theory and by comparing them with the chiral primary operators of the SCFT on the boundary. There exist also $\mathcal{N} = 2, 1, 0$ superconformal theories in 4 dimensions which have corresponding supergravity description on orbifolds of $AdS_5 \times \mathbf{S}^5$. The field theory/M theory duality also provides a supergravity description on AdS_4 or AdS_7 for some superconformal theories in 3 or 6 dimensions. The maximally supersymmetric theories and the lower supersymmetric cases were also realized on the worldvolume of M theory at orbifold singularities.

The gauge group of the boundary theory becomes $SO(N)/Sp(2N)$ [4] by taking appropriate orientifold operations for the string theory on $AdS_5 \times \mathbf{S}^5$. According to general arguments in [4], Type IIB string theory on $AdS_5 \times X_5$ where X_5 is a five dimensional Einstein manifold with five-form flux is dual to a four dimensional SCFT. In [5] it was found that for the $X_5 = (SU(2) \times SU(2))/U(1)$, the string theory on $AdS_5 \times X_5$ can be described by $\mathcal{N} = 1$ supersymmetric $SU(N) \times SU(N)$ gauge theories coupled to four bifundamental chiral superfields and supplemented by a quartic superpotential. A field theory analysis of anomalous three point function reproduced [6] the central charge expected by supergravity. Baryon like chiral operators [7] made out of N chiral superfields were identified with D3 branes wrapped over the 3-cycle of X_5 and domain wall in AdS_5 was interpreted as D5 brane wrapped over 2-cycle of X_5 . Moreover the full KK spectrum analysis was done in a series of paper [8, 9].

By generalizing the work of [4] to the case of $AdS_7 \times \mathbf{RP}^4$ where the eleventh dimensional circle is one of AdS_7 coordinates, $(0, 2)$ six dimensional SCFT on a circle rather than uncompactified full M theory was described in [10]. For $SU(N)$ $(0, 2)$ theory, a wrapped D4 brane on \mathbf{S}^4 together with fundamental strings was interpreted as baryon vertex. Furthermore 3 dimensional extension [11] was obtained by considering D6 branes wrapping on \mathbf{RP}^6 . Backgrounds of the form $AdS_4 \times X_7$ arise as the near horizon geometry of a collection of M2 branes in M theory [12]. Many examples where X_7 is a coset manifold G/H were studied in the old days of KK theories. It is natural to ask *what is dual superconformal field theory corresponding to M theory on $AdS_4 \times X_7$?* As a first step, we will consider only $X_7 = Q^{1,1,1}$ in this paper. Recently the dual theory corresponding to this specific compactification in [13] turns out to be a nontrivial infrared fixed point. It is the $SU(N) \times SU(N) \times SU(N)$ gauge theories with three series of chiral fields $A_i, i = 1, 2$ transforming in the $(\mathbf{N}, \overline{\mathbf{N}}, \mathbf{1})$ representation, $B_j, j = 1, 2$ transforming

in the $(\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}})$ representation and $C_k, k = 1, 2$ transforming in the $(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N})$ representation.

¹ The global symmetry of the gauge theory is $SU(2) \times SU(2) \times SU(2)$ where each of doublets of chiral fields transforms in the fundamental representation of one of the $SU(2)$'s.

Since $Q^{1,1,1}$ described as the coset spaces G/H where $G = SU(2) \times SU(2) \times SU(2)$ and $H = U(1) \times U(1)$ has the isometry $SU(2) \times SU(2) \times SU(2) \times U(1)$ [16], we are looking for an isolated singularity Calabi-Yau fourfold with this symmetry. That is, the geometry of eight dimensional cone is Calabi-Yau fourfold while that of seven dimensional X_7 is Sasaki-Einstein manifold [12, 17]. ² Let us consider the complex manifold C ,

$$z_0^2 + z_1^2 + z_2^2 + z_3^2 + z_4^2 + z_5^2 + z_6^2 + z_7^2 = 0 \quad (1)$$

on \mathbf{C}^8 . This equation describes a surface which is smooth apart from the origin. The apex or node is a double point, i.e. a singularity for which $C = 0$ and $dC = 0$ but for which the matrix of second derivatives is nondegenerate. Note that if z_i solves (1) so does λz_i for any λ , so the surface is made of complex lines through the origin and is a cone. The base of the cone is given by the intersection of the space of solution of (1) with a sphere of radius 1 in \mathbf{C}^8 ,

$$|z_0|^2 + |z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 + |z_5|^2 + |z_6|^2 + |z_7|^2 = 1. \quad (2)$$

These eight coordinates, invariant \mathbf{C}^* action described by toric geometry [15] can be expressed by triple product of six chiral fields A_i, B_j, C_k with some independent embedding equations. It turns out that two D term equations modded out by the action of two $U(1)$'s give exactly the manifold $(\mathbf{S}^3 \times \mathbf{S}^3 \times \mathbf{S}^3)/(U(1) \times U(1))$. The metric on the conifold may be written as

$$ds^2 = dr^2 + r^2 g_{ij} dx^i dx^j, \quad (i, j = 1, 2, \dots, 7) \quad (3)$$

where g_{ij} is the metric on the base of the cone that is exactly $Q^{1,1,1}$. The radial coordinate r is identified with the fourth coordinate of AdS_4 and the section of the cone is identified with the internal manifold $Q^{1,1,1}$. See also many papers [19, 20, 21, 22, 23, 24, 25] dealt with conifold singularity.

In this paper, in section 2, we recapitulate the spectrum of scalar Laplacian on $Q^{1,1,1}$ found in [26] sometime ago, using the metric and seven coordinates of $Q^{1,1,1}$ explicitly. The hypermultiplet spectrum in KK harmonic expansion on $Q^{1,1,1}$ agrees with the chiral fields predicted by dual conformal field theory as shown by [13]. In section 3, we clarify the property of three baryon like operators identified as M5 branes wrapped around 5-cycle. In section 4, we claim that M5 brane wrapped over 3-cycle of $Q^{1,1,1}$ plays the role of domain wall in AdS_4 and explain corresponding dual field theory when the baryon like operators cross the domain wall. Finally we will come to remaining wrapped branes.

¹A different dual SCFT proposal was made in [14, 15].

²The minimal supersymmetric $\mathcal{N} = 1$ CFT analysis and its RG flow have been studied when M2 branes are located at the conical singularity on eight dimensional manifold with $Spin(7)$ holonomy in [18].

2 Laplacian of $Q^{1,1,1}$ and Spectrum of Chiral Operators

To describe the spectrum of chiral primaries in the Q^{n_1, n_2, n_3} CFT we need the expression for scalar Laplacian on Q^{n_1, n_2, n_3} . Since $X_7 = Q^{n_1, n_2, n_3}$ is a $U(1)$ bundle over $\mathbf{S}^2 \times \mathbf{S}^2 \times \mathbf{S}^2$, we take the spherical polar coordinates (θ_i, ϕ_i) , $i = 1, 2, 3$ to parametrize i -th two sphere, as usual, and the angle ψ parametrizes the $U(1)$ fiber. By inverse Kaluza-Klein method, the seven dimensional metric consists of $U(1)$ fiber coordinate together with a potential and six dimensional base $\mathbf{S}^2 \times \mathbf{S}^2 \times \mathbf{S}^2$. From the most general expression for harmonic two-form $U(1)$ field strength, the metric on Q^{n_1, n_2, n_3} is given by [27, 28, 29]

$$g_{ij}dx^i dx^j = c^2 \left(d\psi + \sum_{i=1}^3 n_i \cos \theta_i d\phi_i \right)^2 + \sum_{i=1}^3 \frac{1}{\Lambda_i} \left(d\theta_i^2 + \sin^2 \theta_i d\phi_i^2 \right) \quad (4)$$

where c is a constant, Λ_i are scaling factors and n_i 's characterize the winding numbers of the $U(1)$ field over three \mathbf{S}^2 's. The angles vary over the ranges

$$\theta_i \in (0, \pi), \quad \phi_i \in (0, 2\pi), \quad \psi \in (0, 4\pi). \quad (5)$$

All n_i 's must be integers because we assigned period 4π on the variable ψ . Notice that all three integers n_i 's are necessary to characterize these spaces (while M^{pqr} space can be characterized by only two integers). One obtains inverse metric $g^{i,j}$ from (4) and the nonzero components are

$$\begin{aligned} g^{0,0} &= \frac{1}{c^2} + \sum_{i=1}^3 \Lambda_i n_i^2 \cot^2 \theta_i, \quad g^{2i-1, 2i-1} = \Lambda_i, \quad g^{2i, 2i} = \Lambda_i \csc^2 \theta_i, \\ g^{0, 2i} &= g^{2i, 0} = -\Lambda_i n_i \cot \theta_i \csc \theta_i, \quad i = 1, 2, 3. \end{aligned} \quad (6)$$

We get the volume of Q^{n_1, n_2, n_3} by integrating $\sqrt{g} = \sqrt{\det g_{ij}} = c \sin \theta_1 \sin \theta_2 \sin \theta_3 / (\Lambda_1 \Lambda_2 \Lambda_3)$ over the allowed range of variables,

$$\text{Vol}(Q^{n_1, n_2, n_3}) = 256\pi^4 \frac{c}{\Lambda_1 \Lambda_2 \Lambda_3}. \quad (7)$$

By using the seven coordinates, determinant g and inverse metric components explicitly, the Laplacian can be expressed as

$$\begin{aligned} \square \Phi &= \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} g^{ij} \sqrt{g} \frac{\partial}{\partial x^j} \Phi \\ &= \left(\frac{1}{c^2} \frac{\partial^2}{\partial \psi^2} + \sum_{i=1}^3 \Lambda_i \left(n_i \cot \theta_i \frac{\partial}{\partial \psi} - \csc \theta_i \frac{\partial}{\partial \phi_i} \right)^2 + \sum_{i=1}^3 \frac{1}{\sin \theta_i} \frac{\partial}{\partial \theta_i} \Lambda_i \sin \theta_i \frac{\partial}{\partial \theta_i} \right) \Phi \\ &= -E\Phi. \end{aligned} \quad (8)$$

This can be solved by separation of variables eventhough the Q^{n_1, n_2, n_3} is not a product space. By writing

$$\Phi = \left(\prod_{i=1}^3 \Phi_i(\theta_i) \right) \exp \left(i \sum_{i=1}^3 m_i \phi_i \right) \exp(is\psi), \quad (9)$$

we get

$$E = \sum_{i=1}^3 \Lambda_i E_i + \frac{s^2}{c^2} \quad (10)$$

where E_i 's satisfy the ordinary differential equations

$$\left(\frac{1}{\sin\theta_i} \frac{\partial}{\partial\theta_i} \sin\theta_i \frac{\partial}{\partial\theta_i} - (sn_i \cot\theta_i - m_i \csc\theta_i)^2 \right) \Phi_i = -E_i \Phi_i. \quad (11)$$

Notice that for $n_i = 0$ and $n_i = 1$ (11) determines the eigenvalues of the Laplacian on \mathbf{S}^2 and \mathbf{S}^3 respectively. By defining $z_i = \cos^2 \frac{\theta_i}{2}$, it is easy to see that (11) becomes a hypergeometric equation with a solution

$$\Phi_i = z_i^I (1 - z_i)^J F(A, B, C; z_i) \quad (12)$$

where A, B, C, I, J are smooth in the interval $\theta_i \in (0, \pi)$ and have a behavior at the end points that can be determined by usual formula ³. The solutions are regular when they can be expressed in terms of a hypergeometric polynomials. It is known that ⁴ this happens when

$$\frac{1}{2} - \sqrt{\frac{1}{4} + E_i + |sn_i|^2 + \max(|sn_i|, |m_i|)} = 0, -1, -2, \dots \quad (13)$$

By writing $l_i = k_i + \max(|sn_i|, |m_i|)$ where $k_i = 0, 1, 2, \dots$ we get $E_i = l_i(l_i + 1) - n_i^2 s^2$. Therefore the eigenvalue of the Laplacian on Q^{n_1, n_2, n_3} from (10) is given by

$$E = \sum_{i=1}^3 \Lambda_i \left(l_i(l_i + 1) - n_i^2 s^2 \right) + \frac{s^2}{c^2} \quad (14)$$

where $l_i = |n_i s|, |n_i s| + 1, \dots$. The eigenvalue E is classified by $U(1)$ charge s and spins l_i 's under $SU(2) \times SU(2) \times SU(2)$. The eigenmodes occur in the $(\mathbf{2l_1} + \mathbf{1}, \mathbf{2l_2} + \mathbf{1}, \mathbf{2l_3} + \mathbf{1})$ dimensional representation of $SU(2) \times SU(2) \times SU(2)$ with $U(1)$ charges $s = 0, \pm \frac{1}{2}, \pm 1, \dots$. The eigenvalues (14) as a linear combination of the quadratic Casimirs for the symmetry group $SU(2) \times SU(2) \times SU(2) \times U(1)$ are the form for a coset manifold [26] sometime ago.

The dimension of the scalar operator in terms of energy labels, in the dual SCFT corresponding $AdS_4 \times Q^{1,1,1}$ is

$$\Delta = \frac{3}{2} + \frac{1}{2} \sqrt{1 + \frac{m^2}{4}} = \frac{3}{2} + \frac{1}{2} \sqrt{45 + \frac{E}{4} - 6\sqrt{36 + E}}. \quad (15)$$

³It is well known in any mathematical tables that $F(A, B, C; z_i) = \frac{\Gamma(C)\Gamma(C-A-B)}{\Gamma(C-A)\Gamma(C-B)} F(A, B; A+B-C+1; 1-z_i) + (1-z_i)^{C-A-B} \frac{\Gamma(C)\Gamma(A+B-C)}{\Gamma(A)\Gamma(B)} F(C-A, C-B; C-A-B+1; 1-z_i)$.

⁴It turns out that $A = \frac{1}{2} + m_i - \sqrt{\frac{1}{4} + E_i + |sn_i|^2}$, $B = \frac{1}{2} + m_i + \sqrt{\frac{1}{4} + E_i + |sn_i|^2}$ and $C = 1 + m_i - sn_i$.

The first equation in (15) comes from the relation between the lowest energy eigenvalue and the mass which appears in the AdS_4 wave equation. The relation of d'Alembertian in AdS_4 to Casimir operator was obtained in [30, 31]. For the scalars, $\Delta = \frac{3}{2} + \frac{1}{2}\sqrt{1 + m^2/4}$. The second relation in (15) comes from the formula of mass $m^2 = E + 176 - 24\sqrt{E + 36}$ in [32] where the normalization for this is $(\square - 32 + m^2)S = 0$ for scalar field S . This can be read off directly from the eigenvalues of \square because the relating fields have a mode expansion in terms of the scalar eigenfunctions on Q^{n_1, n_2, n_3} . Other bosonic spectrum and fermionic spectrum can be obtained by calculating higher spin Laplacian also which are more complicated. See also [33]. Although the spectrum of dimensions on all the Q^{n_1, n_2, n_3} is not much interested in, $Q^{1,1,1}$ exhibits an interesting feature which is relevant to superconformal algebra. In this case, the Einstein condition implies that $\Lambda_i = 4\Lambda/3$ and $1/c^2 = 8\Lambda/3$ (coming from the explicit form of Ricci tensor [28, 29] which we did not write down) and hence from (14)

$$E(Q^{1,1,1}) = \frac{4}{3}\Lambda \left(\sum_{i=1}^3 l_i(l_i + 1) - s^2 \right) \quad (16)$$

where $l_i \geq |s|$, $s = 0, \pm\frac{1}{2}, \pm 1, \dots$. The $U(1)$ part of the isometry group of $Q^{1,1,1}$ acts by shifting s . The integer R charge, R is related to s by $s = R/2$. Let us take $R \geq 0$. One can find the lowest value of Δ is R and corresponds to a mode scalar with $l_i = s$ because $E(Q^{1,1,1})$ becomes $32(2s^2 + 3s)$ with $\Lambda = 24$ and plugging back to (15) then $\Delta = R$. Thus we find a set of operators filling out a $(\mathbf{R} + \mathbf{1}, \mathbf{R} + \mathbf{1}, \mathbf{R} + \mathbf{1})_{\mathbf{R}/2}$ multiplet of $SU(2) \times SU(2) \times SU(2) \times U(1)$ where a subscript is $U(1)$ charge $R/2$ and the number $\mathbf{R} + \mathbf{1}$ is the dimension of each $SU(2)$ representation. $\Delta = R$ saturates the bound on Δ from superconformal algebra. It was shown in [13] recently that from the harmonic analysis on $Q^{1,1,1}$ and the spectrum of $SU(2) \times SU(2) \times SU(2)$ representation of the $OSp(2|4)$ hypermultiplets, the hypermultiplet of conformal dimension $\Delta = R$ and $U(1)$ charge $s = R/2$ should be in the representation $l_i = s = R/2$.⁵

According to [34], the information on the Laplacian eigenvalues allows us to get the spectrum of hypermultiplets of the theory corresponding to the chiral operators of the CFT. This part of spectrum was given in [13] and the form of operators is $\text{Tr}(ABC)^R$ where the $SU(2)$'s indices are totally symmetrized. From this, the dimension of ABC should be 1. Although the complete

⁵As pointed out in [25], there is a subtlety for finding the correct conformal dimension among two roots, Δ_+ which we denoted as simply Δ in (15) and Δ_- with minus sign, in the supergravity side. So far we assumed that the integer number R is greater than 1. So the square root of first equation in (15) gives rise to $2R - 3$. However, when $R = 1$, this expression goes like $3 - 2R$. Therefore, we have to choose Δ_- in order to get the correct conformal dimension which is equal to R . So in our case, this is another example of the AdS/CFT duality where the unconventional Δ_- branch has to be chosen for the operators which has R charge of 1. Recall that the \mathbf{S}^7 case where the spherical harmonics correspond to traceless symmetric tensors of $SO(8)$. All chiral operators in the $\mathcal{N} = 8$ $SU(N)$ theory correspond to the conventional branch of dimension Δ_+ except just one case. It is well known that this family of operators with dimension $\Delta = k/2$, $k = 2, 3, \dots$ is $\text{Tr} X^{i_1} X^{i_2} \dots X^{i_k}$ where X^i are the scalars in the vector multiplet. Using $\Delta(\Delta - 3) = m^2$, it is easy to see that $\Delta = k/2 = \Delta_+$, $k = 3, 4, \dots$ and $\Delta = k/2 = \Delta_-$, $k = 2$.

KK spectrum is not known yet, we expect that the relevant operators in higher towers are descendant fields of $\text{Tr}(ABC)^R$ which would have the form of $\text{Tr}F_1^2(ABC)^R + \text{Tr}F_2^2(BCA)^R + \text{Tr}F_3^2(CAB)^R$. Although the dimension of nonchiral operators are in general irrational, there exist special integer values of k_i such that for $l_i = k_i + s$, the Diophantine like condition, $-2(k_1k_2 + k_2k_3 + k_3k_1) + \sum_{i=1}^3(k_i^2 - k_i) = 0$ make $\sqrt{36 + E}$ be equal to $8s + 2(2\sum_{i=1}^3 k_i + 3)$. Furthermore in order to make the dimension be rational, $45 + E/4 - 6\sqrt{36 + E}$ should be square of something. It turns out this is the case without any further restrictions on k_i 's. Therefore we have $\Delta = R + \sum_i^3 k_i$ which is Δ_+ for $\Delta \geq 3/2$ and Δ_- for $\Delta \leq 3/2$. Now we list some operators whose conformal dimensions are integers in terms of their representation $(2\mathbf{l}_1 + 1, 2\mathbf{l}_2 + 1, 2\mathbf{l}_3 + 1)_s$.

$$\begin{aligned}\Delta_- = 1 : & (\mathbf{1}, \mathbf{1}, \mathbf{3})_0, (\mathbf{1}, \mathbf{3}, \mathbf{1})_0, (\mathbf{3}, \mathbf{1}, \mathbf{1})_0, (\mathbf{2}, \mathbf{2}, \mathbf{2})_{\pm 1/2}, \\ \Delta_+ = 2 : & (\mathbf{2}, \mathbf{2}, \mathbf{4})_{\pm 1/2}, (\mathbf{2}, \mathbf{4}, \mathbf{2})_{\pm 1/2}, (\mathbf{4}, \mathbf{2}, \mathbf{2})_{\pm 1/2}, (\mathbf{3}, \mathbf{3}, \mathbf{3})_{\pm 1}, \\ \Delta_+ = 3 : & (\mathbf{3}, \mathbf{3}, \mathbf{5})_{\pm 1}, (\mathbf{3}, \mathbf{5}, \mathbf{3})_{\pm 1}, (\mathbf{5}, \mathbf{3}, \mathbf{3})_{\pm 1}, (\mathbf{4}, \mathbf{4}, \mathbf{4})_{\pm 3/2}.\end{aligned}\tag{17}$$

From the discussion of [26], the first series($\Delta = 1$) give rise to possess extra massless supermultiplets while from the second series($\Delta = 2$) there are additional massless 0^+ in massive supermultiplets. The supermultiplet containing $(\mathbf{2}, \mathbf{2}, \mathbf{2})_{\pm 1/2}$ has to include another scalars and one of them corresponds to the lower component of the superfield $\text{Tr}(ABC)$ which has dimension $\Delta_- = 1$ while other corresponds to the upper component which has dimension $\Delta_+ = 2$. Therefore supersymmetry requires that one chooses dimension Δ_+ for one scalar and Δ_- for the other. It is easy to check that the value of $45 + E/4 - 6\sqrt{36 + E}$ is greater than equal to 0 for all possible values of E . There are no states below the Breitenlohner-Freedman bound [35]. It is possible to have the second solution with minus sign in front of square root in (15) provided the conformal dimension Δ is greater than or equal to $1/2$ which is a unitary bound. Whether descendant fields whose dimensions are larger than the dimension of its chiral primary parent are protected or not will be clear when one understands the full supergravity solution.

3 Baryon like Operators

By putting a large number of N of coincident M2 branes at the conifold singularity and taking the near horizon limit, the metric becomes that [36, 13] of $AdS_4 \times Q^{1,1,1}$

$$ds_{11}^2 = \frac{r^4}{L^{2/3}} \eta_{\mu\nu} dy^\mu dy^\nu + L^{1/3} \left(\frac{dr^2}{r^2} + g_{ij} dx^i dx^j \right).\tag{18}$$

The scale L is related to N by [13]

$$L = \left(\frac{\Lambda}{6} \right)^{-3} = \ell_p^6 2^5 \pi^2 N \frac{\text{Vol}(\mathbf{S}^7)}{\text{Vol}(Q^{1,1,1})}\tag{19}$$

where ℓ_p is a Planck scale which is the only universal parameter in M theory and $\text{Vol}(\mathbf{S}^7) = \pi^4(6/\Lambda)^{7/2}/3$.⁶ The first equation arises when we write AdS_4 radius in terms of both cosmological constant Λ and scale factor L . Since M2 branes have the operators with dimension \sqrt{N} by M2 tension formula and M5 branes have the operators with dimension N through the relation between mass, tension [37] and volume of branes, we consider wrapping a M5 brane over 5-cycle of $Q^{1,1,1}$. Three 5-cycles spanning $H_5(Q^{1,1,1})$ are the restrictions of the $U(1)$ fibration to the product of two of the three \mathbf{P}^1 's. A 5-cycle of minimum volume is to take the subspace at a constant value of (θ_3, ϕ_3) in the metric (4). To calculate the 5 volume, $\text{Vol}(5\text{-cycle})$, it is necessary to find the determinant of the following metric by taking the subspace at a constant value of, for example, (θ_3, ϕ_3) in the metric (4)

$$\frac{3}{8\Lambda} \left(d\psi + \sum_{i=1}^2 \cos \theta_i d\phi_i \right)^2 + \frac{3}{4\Lambda} \sum_{i=1}^2 \left(d\theta_i^2 + \sin^2 \theta_i d\phi_i^2 \right). \quad (20)$$

By integrating the square root of the determinant over the five coordinates, one can find

$$\text{Vol}(5\text{-cycle}) = \frac{\pi^3}{4} \left(\frac{6}{\Lambda} \right)^{5/2}. \quad (21)$$

Other two 5-cycles can be obtained by changing the role of three \mathbf{P}^1 's and it turns out that their volumes are the same. The mass of the M5 brane wrapped over 5-cycle, given by M5 brane tension times $\text{Vol}(5\text{-cycle})$, is

$$m = \frac{1}{(2\pi)^5 \ell_p^6} \text{Vol}(5\text{-cycle}). \quad (22)$$

By the relation (15)

$$m^2 = \frac{2\Lambda}{3}(\Delta - 1)(\Delta - 2) \approx \frac{2\Lambda}{3}\Delta^2 \quad (23)$$

for large Δ and the relations (22) and (19), one gets for the mass formula [13] for the dimension of a baryon corresponding to the M5 brane wrapped 5-cycle

$$\Delta = \frac{\pi N}{\Lambda} \frac{\text{Vol}(5\text{-cycle})}{\text{Vol}(Q^{1,1,1})} = \frac{N}{3} \quad (24)$$

where the volume of $Q^{1,1,1}$ is $\text{Vol}(Q^{1,1,1}) = \frac{\pi^4}{8} \left(\frac{6}{\Lambda} \right)^{7/2}$, given by (7).

Next thing we do is to find corresponding operators in dual field theory whose dimension is N . Since the fields $A_{k\beta}^\alpha$ carry an index α in the \mathbf{N} of $SU(N)_1$ and an index β in the $\overline{\mathbf{N}}$ of

⁶The normalization [13] for four-form field strength is $G_{ijkl} = e\epsilon_{ijkl}$ where the parameter e is a real constant. By plugging this into the 11 dimensional field equations, it leads to the product of 4 dimensional Einstein space, $R_{\mu\nu} = -2\Lambda\eta_{\mu\nu}$ with Minkowski signature $(-, +, +, +)$ and 7 dimensional Einstein space $R_{ij} = \Lambda g_{ij}$ where Λ is defined by $\Lambda = 24e^2/\kappa^{4/9}$ through gravitational constant κ . Moreover $\kappa^2 = 8\pi G_{11} = (2\pi)^8 \ell_p^9/2$.

$SU(N)_2$, one can construct a baryon like color singlet operator by antisymmetrizing completely with respect to both groups. The resulting gauge invariant chiral operator is

$$\mathcal{B}_{1l} = \epsilon_{\alpha_1 \dots \alpha_N} \epsilon^{\beta_1 \dots \beta_N} D_l^{k_1 \dots k_N} \prod_{i=1}^N A_{k_i \beta_i}^{\alpha_i} \quad (25)$$

where $D_l^{k_1 \dots k_N}$ is the completely symmetric $SU(2)$ Clebsch-Gordon coefficient corresponding to forming the $\mathbf{N} + \mathbf{1}$ of $SU(2)$ out of N $\mathbf{2}$'s. Therefore, the $SU(2) \times SU(2) \times SU(2)$ quantum numbers of \mathcal{B}_{1l} are $(\mathbf{N} + \mathbf{1}, \mathbf{1}, \mathbf{1})$. In order to understand these $SU(2)$ quantum numbers, it is necessary to do collective coordinate quantization of the wrapped M5 brane along the five coordinates $(\psi, \theta_2, \phi_2, \theta_3, \phi_3)$ which acts as a charged particle because nonzero Betti number of $Q^{1,1,1}$ implies nonperturbative states only which can be charged. Since the lowest angular momentum of a charge particle is $N/2$, the ground state collective coordinate wave functions form a $\mathbf{N} + \mathbf{1}$ dimensional representation of the first $SU(2)$ which rotates the first \mathbf{S}^2 . This \mathbf{S}^2 is not wrapped by M5 brane because it is localized at a constant coordinate (θ_1, ϕ_1) . Of course, wrapped M5 brane is a singlet under other $SU(2)$'s.

Similarly, one can construct baryon like operators which transform as $(\mathbf{1}, \mathbf{N} + \mathbf{1}, \mathbf{1})$,

$$\mathcal{B}_{2l} = \epsilon_{\beta_1 \dots \beta_N} \epsilon^{\gamma_1 \dots \gamma_N} D_l^{k_1 \dots k_N} \prod_{i=1}^N B_{k_i \gamma_i}^{\beta_i}. \quad (26)$$

The $SU(2) \times SU(2) \times SU(2)$ quantum numbers of \mathcal{B}_{2l} in this case are $(\mathbf{1}, \mathbf{N} + \mathbf{1}, \mathbf{1})$. The fields $B_{k\gamma}^\beta$ carry an index β in the \mathbf{N} of $SU(N)_2$ and an index γ in the $\overline{\mathbf{N}}$ of $SU(N)_3$. The ground state collective coordinate wave functions represent a $\mathbf{N} + \mathbf{1}$ dimensional representation of the second $SU(2)$ which rotates the second \mathbf{S}^2 . Finally baryon like operators which transform as $(\mathbf{1}, \mathbf{1}, \mathbf{N} + \mathbf{1})$ are

$$\mathcal{B}_{3l} = \epsilon_{\gamma_1 \dots \gamma_N} \epsilon^{\alpha_1 \dots \alpha_N} D_l^{k_1 \dots k_N} \prod_{i=1}^N C_{k_i \alpha_i}^{\gamma_i}. \quad (27)$$

Here the fields $C_{k\alpha}^\gamma$ carry an index γ in the \mathbf{N} of $SU(N)_3$ and an index α in the $\overline{\mathbf{N}}$ of $SU(N)_1$. The ground state collective coordinate wave functions represent a $\mathbf{N} + \mathbf{1}$ dimensional representation of the third $SU(2)$ which rotates the third \mathbf{S}^2 . Under the symmetry which exchanges the fundamental fields A, B, C of the gauge groups $SU(N)_1 \times SU(N)_2 \times SU(N)_3$, these operators map to M5 branes localized at either constant (θ_2, ϕ_2) or (θ_3, ϕ_3) . The existence of three types of baryon operators is related to the fact that the base of $U(1)$ bundle of internal space is $\mathbf{S}^2 \times \mathbf{S}^2 \times \mathbf{S}^2$. Since each of A, B, C has dimension $1/3$ in the construction [13] due to the fact that we have seen the conformal dimension of ABC is 1 in the previous section and a permutation symmetry among them, the dimension of the baryon like operators is $N/3$ which is in agreement with supergravity calculation we have worked before (24). This implies that *three 5-cycles are supergravity representations of conformal operators (25), (26) and (27).*

For consistency check, one can consider the dimension of Pfaffian operator in $SO(2N)$ gauge theory. Gauge invariant baryonic operator $\epsilon_{a_1 \dots a_{2N}} \Phi^{a_1 a_2} \dots \Phi^{a_{2N-1} a_{2N}}$ has dimension $N/2$. The $SO(2N)$ theory is dual to $AdS_4 \times \mathbf{RP}^7$ and the dual Pfaffian wrapping M5 brane on a \mathbf{RP}^5 gives, according to (24)

$$\Delta = \frac{\pi N}{\Lambda} \frac{\text{Vol}(5\text{-cycle})}{\text{Vol}(X_7 = \mathbf{RP}^7)} = \frac{\pi N}{\Lambda} \frac{\text{Vol}(\mathbf{RP}^5)}{\text{Vol}(\mathbf{RP}^7)} = \frac{N}{2}. \quad (28)$$

Moreover, an $\mathcal{N} = 2$ theory [38] results from \mathbf{Z}_3 orbifold action on \mathbf{S}^7 defined by coordinatizing \mathbf{R}^6 by three complex numbers z_1, z_2, z_3 orthogonal to M2 brane worldvolume and considering the map $z_k \rightarrow e^{2\pi i/3} z_k$ for all k . Minimal area 5-cycles on $\mathbf{S}^7/\mathbf{Z}_3$ can be constructed by intersecting the 5-plane $z_k = 0$ for any particular k with the sphere $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$. Then

$$\Delta = \frac{\pi N}{\Lambda} \frac{\text{Vol}(5\text{-cycle})}{\text{Vol}(X_7 = \mathbf{S}^7/\mathbf{Z}_3)} = \frac{\pi N}{\Lambda} \frac{\text{Vol}(\mathbf{S}^5)/3}{\text{Vol}(\mathbf{S}^7)/3} = \frac{N}{2}. \quad (29)$$

The theory has gauge group $SU(N)_1 \times SU(N)_2 \times SU(N)_3$ with three $(\mathbf{N}, \overline{\mathbf{N}})$ representations between each pair of gauge groups. Baryon like operators as in (25), (26) or (27) from the bifundamental matter have dimension $N/2$.

4 Domain Walls in AdS_4 and Other Wrapped Branes

Since AdS_4 has three spatial dimensions, M2 branes in AdS_4 could potentially behave as a domain wall. Since M2 brane is the electric source of the four form field G_4 , the integrated hodge dual of four-form flux over 7 manifold $\int_{X_7} \star G_4$ jumps by one unit when one crosses the domain wall. This means the gauge group of the boundary conformal field theory can change, for example, from $SU(N)_1 \times SU(N)_2 \times SU(N)_3$ on one side to $SU(N+1)_1 \times SU(N+1)_2 \times SU(N+1)_3$ on the other side for $AdS_4 \times Q^{1,1,1}$. Of course, for the anti-M2 brane, the gauge group will change $SU(N-1)_1 \times SU(N-1)_2 \times SU(N-1)_3$. The similar situation also occurs when M5 brane is wrapped on a specific 3-cycle of $Q^{1,1,1}$ to make a M2 brane in AdS_4 . Using the orthonormal bases generated by the vielbeins of $Q^{1,1,1}$ for given metric (4)

$$e^\psi = \sqrt{\frac{3}{8\Lambda}} \left(d\psi + \sum_{i=1}^3 \cos \theta_i d\phi_i \right), \quad e^{\theta_i} = \sqrt{\frac{3}{4\Lambda}} d\theta_i, \quad e^{\phi_i} = \sqrt{\frac{3}{4\Lambda}} \sin \theta_i d\phi_i \quad (30)$$

where $i = 1, 2, 3$, the harmonic representatives of second, third and fifth cohomology groups can be written in terms of these combinations. Note that from 4-th cohomology, $H^4(Q^{1,1,1}, \mathbf{Z}) = \mathbf{Z}_2 \cdot (\omega_1 \omega_2 + \omega_1 \omega_3 + \omega_2 \omega_3)$ where ω_i 's are the generators of the second cohomology group of the \mathbf{P}^1 's, 3rd homology $H_3(Q^{1,1,1}, \mathbf{Z})$ can be obtained. In general, 3-cycle can be viewed as a fibration of ψ over 2 sphere parametrized by some combination of (θ_i, ϕ_i) , $i = 1, 2, 3$, but we are thinking a domain wall (a M5 brane wrapped over 3-cycle) together with baryon like operator (a

M5 brane wrapped over 5-cycle). In the previous section, we considered 5-cycle as a fibration of ψ over $(\theta_i, \phi_i, \theta_j, \phi_j), i, j = 1, 2, 3$. We take 3-cycle as 3 dimensional space orthogonal to 5-cycle except one common direction. There exist three ways for M5 brane to be wrapped on 3-cycle depending on the choice of three \mathbf{S}^2 's in the base of $U(1)$ bundle. When on one side of the domain wall one has the original $SU(N)_1 \times SU(N)_2 \times SU(N)_3$, then on the other side the corresponding one is $SU(N)_1 \times SU(N)_2 \times SU(N+1)_3$ if we take first 3-cycle. The matter fields A_i, B_j, C_k are filling out $2(\mathbf{N}, \overline{\mathbf{N}}, \mathbf{1}) \oplus 2(\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}} + \overline{\mathbf{1}}) \oplus 2(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N} + \mathbf{1})$. Similarly, if we take second 3-cycle, on the other side the corresponding gauge group is $SU(N)_1 \times SU(N+1)_2 \times SU(N)_3$. The matter fields A_i, B_j, C_k are filling out $2(\mathbf{N}, \overline{\mathbf{N}} + \overline{\mathbf{1}}, \mathbf{1}) \oplus 2(\mathbf{1}, \mathbf{N} + \mathbf{1}, \overline{\mathbf{N}}) \oplus 2(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N})$. Also if we take third 3-cycle, on the other side the corresponding gauge group is $SU(N+1)_1 \times SU(N)_2 \times SU(N)_3$. The matter fields A_i, B_j, C_k are filling out $2(\mathbf{N} + \mathbf{1}, \overline{\mathbf{N}}, \mathbf{1}) \oplus 2(\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}}) \oplus 2(\overline{\mathbf{N}} + \overline{\mathbf{1}}, \mathbf{1}, \mathbf{N})$.

Let us consider what happens if the baryons(wrapped M5 branes over 5-cycle) cross a domain wall(wrapped M5 brane over 3-cycle). Let us wrap M5 brane around particular 5-cycle which is invariant both under the group $SU(2)_B$ under which the fields B_j transform and the group $SU(2)_C$ under which the fields C_k transform. Then the corresponding state in the $SU(N)_1 \times SU(N)_2 \times SU(N)_3$ field theory is \mathcal{B}_1 of (25). In the $SU(N)_1 \times SU(N+1)_2 \times SU(N)_3$ theory, we have

$$\epsilon_{\alpha_1 \dots \alpha_N} \epsilon^{\beta_1 \dots \beta_{N+1}} \prod_{i=1}^N A_{\beta_i}^{\alpha_i} \quad \text{or} \quad \epsilon_{\alpha_1 \dots \alpha_N} \epsilon^{\beta_1 \dots \beta_{N+1}} \prod_{i=1}^{N+1} A_{\beta_i}^{\alpha_i} \quad (31)$$

where $SU(2)$ indices are omitted. These are either a fundamental of $SU(N+1)_2$ or a fundamental of $SU(N)_1$. This is no longer a singlet because when one antisymmetrizes the color indices on a product of N or $N+1$ bifundamentals of $SU(N)_1 \times SU(N+1)_2 \times SU(N)_3$ there exists one free index. If we have M wrapped M5 branes over 3-cycle rather than a single wrapped M5 brane over 3-cycle, then according to M units of flux, the gauge group will be $SU(N)_1 \times SU(N+M)_2 \times SU(N)_3$.

The wrapped M5 brane must have M2 brane attached to it. The new aspect of the domain wall is that M2 brane must stretch from it to wrapped M5 brane. Recall [39, 40] that two M5 branes with one common direction cross, a M2 brane stretched between them is created. By dimensional reduction to the Type IIA string theory one can find T dual version of Hanany Witten effect [41]: When a NS5 brane and a D5 brane sharing two common directions pass through each other, a D3 brane must be created. The action containing a Chern Simons term is proportional to $\int G_4(\theta_2, \phi_2, \theta_3, \phi_3) \wedge B_2(y^0, \psi)$ indicating that G_4 acts as a source of B_2 where $G_4 = dC_3$ is four-form field in M theory and B_2 is a RR B field. From the flux through the baryonic M5 brane along $(y^0, \psi, \theta_2, \phi_2, \theta_3, \phi_3)$ in the presence of domain wall M5 brane along $(y^0, y^2, y^3, \psi, \theta_1, \phi_1)$, *the net charge that couples to B field gives rise to a M2 brane along (y^0, y^1, ψ) stretched between M5 branes is created.* We can reduce to Type IIA string theory

along the ψ , which is common to all branes. Then D4 brane along $(y^0, \theta_2, \phi_2, \theta_3, \phi_3)$ passing D4 brane along $(y^0, y^2, y^3, \theta_1, \phi_1)$ creates a fundamental string along (y^0, y^1) direction.

Similarly, If we consider M5 brane around particular 5-cycle which is invariant both under the group $SU(2)_A$ under which the fields A_i transform and the group $SU(2)_C$ under which the fields C_k transform. Then the corresponding state in the $SU(N)_1 \times SU(N)_2 \times SU(N)_3$ field theory is \mathcal{B}_2 of (26). Then the corresponding $SU(N)_1 \times SU(N)_2 \times SU(N+1)_3$ theory has

$$\epsilon_{\beta_1 \dots \beta_N} \epsilon^{\gamma_1 \dots \gamma_{N+1}} \prod_{i=1}^N B_{\gamma_i}^{\beta_i} \quad \text{or} \quad \epsilon_{\beta_1 \dots \beta_N} \epsilon^{\gamma_1 \dots \gamma_{N+1}} \prod_{i=1}^{N+1} B_{\gamma_i}^{\beta_i}. \quad (32)$$

which become a non singlet because when one antisymmetrizes the color indices on a product of N or $N+1$ bifundamentals of $SU(N)_1 \times SU(N)_2 \times SU(N+1)_3$ there exists one free index. That is, either a fundamental of $SU(N+1)_3$ or a fundamental of $SU(N)_2$. Moreover, when we consider M5 brane over 5-cycle which is invariant both under the group $SU(2)_A$ under which the fields A_i transform and the group $SU(2)_B$ under which the fields B_j transform, the $SU(N)_1 \times SU(N)_2 \times SU(N)_3$ field theory has \mathcal{B}_3 of (27). The corresponding $SU(N+1)_1 \times SU(N)_2 \times SU(N)_3$ theory has

$$\epsilon_{\gamma_1 \dots \gamma_N} \epsilon^{\alpha_1 \dots \alpha_{N+1}} \prod_{i=1}^N C_{\alpha_i}^{\gamma_i} \quad \text{or} \quad \epsilon_{\gamma_1 \dots \gamma_N} \epsilon^{\alpha_1 \dots \alpha_N} \prod_{i=1}^{N+1} C_{\alpha_i}^{\gamma_i}. \quad (33)$$

In this case also there is one free index when one antisymmetrizes the color indices on a product of N or $N+1$ bifundamentals of $SU(N+1)_1 \times SU(N)_2 \times SU(N)_3$.

From $H^3(Q^{1,1,1}, \mathbf{Z}) = H^6(Q^{1,1,1}, \mathbf{Z}) = 0$, there are no states associated with branes wrapping 4-cycle or 1-cycle. For M5 branes, there are three types of wrapping because there exist nonzero $H_5(Q^{1,1,1}, \mathbf{Z})$, $H_3(Q^{1,1,1}, \mathbf{Z})$ and $H_2(Q^{1,1,1}, \mathbf{Z})$. The first case involves 5-cycle and produces particle in AdS_4 associated with baryon like operators (25), (26) and (27). The second case involves 3-cycle and produces a domain wall we have discussed. The last one involves M5 brane wrapping 2-cycle and produces threebrane in AdS_4 . At this moment, we do not know how this can be realized in the full M theory and it is not clear what is interpretation of boundary conformal field theory. When M5 brane is wrapped around the eleventh circle \mathbf{S}^1 orthogonal to 2-cycle, then in Type IIA description, this is equivalent to twobrane in AdS_4 . For M2 branes, there exists only one type of wrapping which involves 2-cycle and produces particle in AdS_4 because M2 brane can not wrap higher dimensional space 5-cycle or 3-cycle. The mass of M2 brane wrapped 2-cycle goes like \sqrt{N} from the mass formula which was not appropriate for the candidate of baryon like operator that must behave like as N .

5 Conclusion

In summary, since $AdS_4 \times Q^{1,1,1}$ is a supersymmetric holographic theory based on a compact manifold $Q^{1,1,1}$ which is not locally S^7 , the dual $SU(N)_1 \times SU(N)_2 \times SU(N)_3$ gauge theory can not be obtained from a projection of the $\mathcal{N} = 8$ theory. The dual representation of baryon like operators from a symmetric product of N bifundamental matter fields, fully antisymmetrized on upper and lower indices separately, is a M5 brane wrapped around an 5-cycle in $Q^{1,1,1}$. Three ways of embedding 5-cycle are orbits of two of the three $SU(2)$ global symmetry groups of the theory. A M5 brane wrapping 5-cycle can be regarded as a charge particle allowed to move on the 2-cycle parametrizing remaining orbits. The 5 volume of the $SU(2)$ orbits gives a dimension for the operators $N/3$ which matches exactly the field theory. By using the baryon like operators, wrapped M5 branes around 3-cycle of $Q^{1,1,1}$ behave as a domain wall separating the original $SU(N)_1 \times SU(N)_2 \times SU(N)_3$ from, for example, $SU(N)_1 \times SU(N+1)_2 \times SU(N)_3$. The crucial point was that a M2 brane is created when a M5 brane(wrapped around 5-cycle) crosses a M5 brane(wrapped around 3-cycle). This means that a baryon is no longer a singlet that agrees with the field theory observation, because there exists one free index when antisymmetrizing the color indices on a product of N or $N+1$ bifundamentals of $SU(N)_1 \times SU(N+1)_2 \times SU(N)_3$.

There are various G/H models which has at least a supersymmetry less than or equal to three in 3 dimensions. It is already known that there exists a dual conformal field theory corresponding to M^{pqr} space. It is natural to ask how other cases can be realized in the boundary conformal field theories. It would be interesting to study whether one can find wrapping branes over various cycles of possible X_7 's and discuss their field theory interpretations. The theory of M2 branes at orbifold singularity, for example, \mathbf{C}^4/Γ tells us a variety of supersymmetric theories [12] depending on how Γ acts. It is not known what is boundary conformal field theory corresponding to M2 branes at $\mathbf{C}^4/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ [42]. It is quite interesting to see how our conifold description can be obtained from a deformation of orbifold singularity.

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